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ORIGINAL PAPER

Stochastic Finite element analysis of the free vibration of functionally graded material plates

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Abstract The superior properties of functionally graded materials (FGM) are usually accompanied by randomness in their properties due to difficulties in tailoring the gradients during manufacturing processes. Using the stochastic finite element method (SFEM) proved to be a powerful tool in studying the sensitivity of the static response of FGM plates to uncertainties in their material properties. This tool is yet to be used in studying free vibration of FGM plates. The aim of this work is to use both a First Order Reliability Method (FORM) and the Second Order Reliability Method (SORM), combined with a nine-noded isoparametric Lagrangian element based on the third order shear deformation theory to investigate sensitivity of the fundamental frequency of FGM plates to material uncertainties. These include the effect of uncertainties on both the metal and ceramic constituents. The basic random variables include ceramic and metal Young's modulus and Poisson's ratio, their densities and ceramic volume fraction. The developed code utilizes MATLAB capabilities to derive the derivatives of the stiffness and mass matrices symbolically with a considerable reduction in calculation time. Calculating the eigenvectors at the mean values of the variables proves to be a reasonable simplification which significantly increases solution speed. The stochastic finite element code is validated using available data in the literature, in addition to comparisons with results of the well-established Monte Carlo simulation technique with importance sampling. Results show that SORM is an excellent rapid tool in the stochastic analysis of free vibration of FGM plates, when compared to the slower Monte Carlo simulation techniques.

Keywords FGM · Stochastic finite element analysis · Shear deformable plate · FORM · SORM

1 Introduction

The superior properties of advanced composite materials, such as high specific strength and high specific stiffness, have led to their widespread use in aircrafts, spacecrafts and space structures. In conventional laminated composite structures, orthotropic elastic laminas are bonded together to obtain enhanced mechanical and thermal properties. However, the abrupt changes in material properties across the interface between different materials can result in large interlaminar stresses leading to delamination. Furthermore, large plastic deformations at the interface may trigger the initiation and propagation of cracks in the material. One way to overcome these adverse effects is to use Functionally Graded Materials (FGM), in which material properties vary continuously. This is achieved, for example, by gradually changing the volume fraction of the constituent materials, usually in the thickness direction only, or by changing the chemical structure of a thin polymer sheet to obtain a smooth variation of in-plane material properties and an optimum response to external thermo mechanical loads.

Due to difficulties in tailoring the gradients to actual specifications during manufacturing processes, properties of FGM's are not deterministic in nature. There is a reasonable

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